On Point Series

On Point is published with the intent to provide institutional investors with insights and to provoke discussion.

In this four part series, we begin by examining Liability Driven Investing (LDI). We then challenge this traditional approach and contrast LDI with the concept of Liability Relative Investing (LRI), and explore it further. The final paper focuses on practical ways to implement LRI while adding alpha to your portfolio.
Back to Basics

Why Pension Funds Exist, and How to Manage Them in the Simplest Possible Way

SUMMARY:
We first ask why pension funds exist at all, in contrast to pay-as-you-go plans. The reason is that a pension is a benefit to employees operating as an obligation. The fund is a buffer creating economic security in the relationship between the sponsor, who makes a promise in one period, and the beneficiary, who needs to be assured that the promise will be fulfilled in a much later period. We then ask how one would manage such a pension fund in the simplest and least risky way that can be achieved using existing assets. We call the resulting strategy liability-driven investing (LDI). The addition of risk in order to earn a higher return (to help pay for the plan) is discussed in future articles in this series, and referred to as liability-relative investing (LRI). We are not advocating actually managing pension funds risklessly, but we will not get anywhere in designing a more complex strategy that allows for risk-taking unless we start at the beginning, which is just to prepay the liability using riskless bonds.

Introduction
It typically takes a real crisis – and the current pension funding crisis is very real – to focus the mind on the idea that, given a clean slate, one would design whole institutions differently. This principle applies particularly to pension funds, which are simple in concept but which have become very complex in practice.

Here’s the simple idea: “If you work for us for [x number of] years, after you stop working we will continue paying you according to some formula until you die.” A pension plan viewed this way is just an income continuation plan. But, in order to get something, a worker (like any economic agent) must give up something of roughly equal value. A pension is therefore often thought of as a deferred compensation plan, since the worker agrees to be paid less while working in order to receive the income continuation after retirement. Whether one thinks of a pension as income continuation or deferred compensation, it doesn’t sound as though setting up such a plan poses much of a challenge — making sure the money is there when needed to pay the retiree is a matter of saving some instead of spending it.

Now, here’s a typical list of implementation techniques: asset-liability modeling; Markowitz optimization; Monte Carlo simulation; manager searches and hiring and firing disciplines; performance measurement, evaluation, and attribution; alpha and beta; a blizzard of new “alternative” asset classes – the list goes on. Clearly the art and science of pension management have taken on a level of complexity out of proportion to the question asked, which is, “How can the need to guarantee the post-retirement payments to employees be balanced against the desire to make these payments at the lowest possible cost?”

Why the complexity? Can we simplify the solution by going back to the problem, appreciating how simple it is, and designing a strategy that fits?

Why a pension fund?
If all plan sponsors – private and governmental – were assured of being financially healthy indefinitely into the future, then pensions could be pay-as-you-go.
plans without exposing the beneficiaries to risk. But companies fail all the time, and governments face the possibility that taxpayers will be unwilling or unable to provide needed funds. So the institution of the pension fund has been set up as a buffer between the sponsor and the beneficiary, to reduce the risk arising from negative events between the time one incurs an obligation and the much later time that the obligation comes due.

A simple plan

Here is a simple plan for making sure that the money is there to pay employees’ pensions or deferred compensation.

- Starting today \((t=0)\), calculate the present value of future benefits (the “liability”) using a riskless (government bond) discount rate.
- Invest that much (i.e. full funding) in riskless government bonds that are duration-matched to the liability. Duration matching will be described in greater detail later.
- Next period \((t=1)\), recalculate the liability, which will have changed due to:
  - Interest rate movements (say you expected a 4% return but you got less than that amount, because interest rates rose and bond prices fell over the period)
  - New hires, deaths, people leaving the plan without vesting, etc.
  - Longevity surprises
  - Anything else that affects the liability
- Make sure the amount invested at time \(t=1\) is equal to the liability, by making a contribution if there is a deficit or by making a withdrawal if there is a surplus.

That’s all there is to it. A pension plan that does what was just described, and nothing more or less, is practicing the purest form of liability-driven investing (LDI). Pension investing really is that simple.

But it is only that simple if the sponsor does not want to take risk in the hope that the markets will help “pay for” the pension benefit. In the real world of tight budgets, fierce competitors, and non-pension executives who are pressured to consider other uses for the money, taking such risks is often irresistibly attractive and necessary. Therein lies a can of worms that, once opened, can only be re-closed using a much larger can – thus the complexity described earlier. The complexity arises because taking risk involves using sophisticated tools for both risk measurement and management.

Future articles in this series will discuss the rewards and hazards of taking market risk, and other kinds of risk, relative to this pure LDI base case or benchmark. This future work will refer to investing with the liability as the benchmark as liability-relative investing, or LRI (following Waring [2004]), to distinguish it from just investing in assets that are both riskless and duration-matched to the liability, which we call LDI. Other managers have other naming conventions.

The Simple Plan Filled Out a Bit

Many say the risk-minimizing investment should be a bond because the pension obligation “looks like a bond.” We will see, however, that it does not.

Figure 1

Note that these curves are intended to be illustrative, and are not exact. The “exact” curves for a given plan would be based on an actuarial study, and even these would change over time as actuarial surprises, such as changes in participant longevity, take place.

Figure 1 shows the cash flows inherent in a typical pension obligation. The reason for the peculiar shape of this cash flow diagram is that pension benefit payments grow rapidly among the population of active workers (those who are currently working) because of inflation,
real increases in salary, and an increase in the number of workers who are vested. Payments then level off and, later, decrease as the covered workers die. Deferred pensioners, those who have stopped working for the company but who may still be working elsewhere, experience a similar pattern but earlier, so that the fastest growth period is in the past (off the chart to the left). Retirees experienced both the growth period and the leveling off in the past (again, off the chart to the left). So, while each company, governmental body, or other pension plan sponsor has its own payout curve, almost all of the curves are a variation on the shape shown in Figure 1.

An ordinary, coupon-paying bond, in contrast, looks like this:

Figure 2

Cash Flows on a 30-Year Bond with 4% Annual Coupon

- Duration = 18 years
- Present value weighted average time to receipt of cash flows from the bond
- Approximate price change of bond per 1% change in interest rate.

If the plan is not fully funded but the duration of the assets is set up so that a parallel shift in the yield curve causes identical dollar changes in the assets and the liability, the plan is still practicing LDI as commonly understood – but we prefer to restrict the use of the LDI terminology to fully-funded plans.

Note that the two cash flow diagrams look nothing alike. A long time ago, pension managers used the principle of cash flow matching to justify building a bond portfolio with cash flows that look like those in Figure 1. Such a result is easily engineered. One need not do so however, because two portfolios with the same duration will have approximately the same price response to a change in the general level of interest rates, even if the cash flows are not matched at all in terms of timing. (Duration is the sensitivity of the present value of a set of cash flows to changes in the overall level of interest rates, and is the principal risk to which pension liabilities are exposed, because the promise to pension beneficiaries is typically structured to provide them with a fixed income after retirement, with or without a cost of living adjustment.)

So, if the cash flows in Figure 1 are owed to the beneficiaries, and the cash flows in Figure 2 are produced by the bond portfolio held as security for the pension promise, there is not much interest rate risk; the portfolios are duration matched (because the cash flows in Figures 1 and 2 have the same duration, 18 years). Note that the match is imperfect. There is residual risk from changes in the slope of the yield curve, and from other factors. In addition, the fund is still exposed to actuarial (workforce population and longevity) risks that cannot be hedged in the capital markets. As we said earlier, a pension plan that is fully funded and that matches the duration of its assets to that of its liabilities, and does nothing else except for making additional contributions (withdrawals) as actuarial surprises cause a deficit (surplus) to arise in the plan, practices LDI.

The most important reason to care about finding the risk-minimizing strategy, by the way, is not that the sponsor wishes to avoid risk although that may well be a consideration, but that by deferring part of their compensation – by working at their jobs anyway, the beneficiaries have earned the right to benefit security! They are just as entitled to their pension as they are to current compensation. Jeopardizing this part of their compensation is morally wrong, legally wrong (the sponsor might be sued), and economically wrong (bad for business, because workers will demand a risk premium in their wages if they think they might not get their earned pension benefits).

Beyond LDI: Taking Risk to Help Pay for the Pension Plan

If LDI produces a more or less satisfactory result, why do plan sponsors instead set up elaborate investment management operations, using the complex tools described at the beginning of this article, and subjecting themselves to all kinds of risks?
The answer is that when you have to hold on to a pile of savings for 20 years or more (see Figure 1), there are real opportunities to earn more than the rate of interest on a bond portfolio. Any return earned above that of the bond portfolio reduces the cost of the pension plan: that is, it reduces the contributions that the sponsor must make to keep the plan fully funded. The equity market presents one such opportunity for earning higher returns. While stocks have not outperformed bonds over every possible 20-year period, they usually have. At any rate, logic dictates that the equity risk premium (that is, the expected return of stocks in excess of bonds) must be a positive number or else equities would not attract many buyers.

Today’s opportunity set of pension investments goes far beyond conventional stocks and bonds and includes pretty much every asset in the world, whether held publicly or privately. The technology and due diligence needed to assess each of the investments, decide whether or not to hold them, and determine how to best combine them in a portfolio requires great sophistication, and represents a tremendous departure from LDI. Most of the intellectual effort that has gone into pension management over the last 30 or 40 years has been devoted to reducing this cost by taking risk optimally.

The modern pension plan, with exposure to equities, alternative assets, and other approaches to return enhancement, has come a long way from the simple plan described earlier. The simple plan took no unnecessary risk, and thus made no attempt to assess the likelihood that taking a given kind and amount of risk would be fruitful. In modern pension management, the balancing of risk against expected return – that is, the assessment of just this question, whether or not taking a given risk is a good idea – is absolutely central. The pension manager is focused not on minimizing risk, but on maximizing return subject to a concern about risk – a very different matter. Markowitz optimization is one way to express this risk-return tradeoff; liability-relative investing (LRI), using the liability as the benchmark, is another. LRI is introduced in greater detail in the next issue of On Point.

The key unknown in actually reducing pension cost is, of course, whether this expected excess return is realized. Most of the current pension crisis comes from having budgeted for pension contributions as though equities and other risky asset classes would automatically provide returns superior to those of bonds. The sad fact is that the markets delivered very disappointing returns over the last 12 years or so. While a repeat of this miserable performance is not our forecast for the future, it could happen again and investors should be prepared for such an event.

The issues involved in taking risk in a pension plan, instead of investing almost risklessly in an LDI strategy, could fill a thick book (and there are many such books). The most important of these issues will be addressed in the remaining essays in this series. In the next issue of On Point, we flesh out the concept of LRI and show how it works. We note the advantages of this approach and describe some applications, and we point out special considerations that arise in applying LRI techniques to non-pension asset pools.


The information provided herein does not constitute a solicitation of an offer to buy, or an offer to sell securities nor should the information be relied upon as investment advice. Past performance is no guarantee of future results.

All Rights Reserved. The information contained herein: (1) is confidential and proprietary to BMO Asset Management Inc. (“BMO AM”); (2) may not be reproduced or distributed without the prior written consent of BMO AM; and (3) has been obtained from third party sources believed to be reliable but which have not been independently verified. BMO AM and its affiliates do not warrant or make any representations regarding the use or the results of the information contained herein in terms of its correctness, accuracy, timeliness, reliability or otherwise, and do not accept any responsibility for any loss or damage that results from its use.

© Registered trade-mark/trade-marks of Bank of Montreal, used under license.
Beyond the Basics

Pension Management using Liability-Relative Investing

**SUMMARY:**
In this paper we contrast liability-driven investing (LDI) with liability-relative investing (LRI). In our previous paper, we defined LDI as investing to hedge away, as completely as possible, the market-related (stock market, bond market, real return bond) risks in the liability. Liability-relative investing uses LDI as a point of departure and then takes risks to earn a higher return. To accomplish this, we start with the mean-variance optimization (MVO) technique pioneered by Harry Markowitz. LRI can be understood in this context as MVO with the liability treated as an asset held short. The optimizer then overweight liability hedges, such as bonds and real return bonds (RRBs), relative to what they would be in the absence of a liability, and underweights cash. Our innovation, drawing on Waring and Whitney (2009), is to note that, for an investor with a liability, the riskless portfolio is not cash. It is the portfolio of bonds and RRBs that is the best hedge to the liability; that is, it’s the LDI portfolio. Risky assets can then be added without sacrificing the hedging properties of the riskless asset.

Our next paper will say how to determine the composition of the LRI portfolio using simplifying assumptions — the Capital Asset Pricing Model.

**Liability-driven and liability-relative investing defined**
In our previous paper, “Back to Basics,” we defined liability-driven investing (LDI) as holding a portfolio that hedges, as fully as possible, the risks to a pension plan or other asset pool caused by changes in the present value of the liability. LDI usually takes the form of a laddered portfolio of bonds and/or RRBs that is duration-matched to the liability.

We then noted that investors can, and usually do, take various kinds of market risk to try to earn a higher return than is available on bonds and RRBs. Such a strategy requires a benchmark. If an investor uses the return on the liability as the benchmark, measuring the results of the investment program by the extent to which the asset return exceeds or falls short of the liability return, then he or she is engaging in liability-relative investing, LRI.

The concept of the “return on the liability” may not be familiar to everyone. It is the return on a hypothetical portfolio of actual, holdable assets that is the best fit or best hedge to the liability. We advocate LRI as the preferred way of thinking about pension investment management, and about the management of certain other types of funds that have a liability.

This paper frames LRI as a Markowitz optimization problem. We arrive at a solution in two steps. The conceptual solution — a general approach to determining a portfolio that hedges the liability and that also takes market risk in pursuit of return — is described in this paper. A practical solution, covered in a sequel paper, involves finding out what assets are held in that portfolio, and at what weights. To arrive at the practical solution requires making some simplifying assumptions. The sequel paper, “The Liability-Relative Solution,” will discuss these simplifying assumptions and show how to arrive at the asset-class weights. Finally, our fourth paper in this series will cover implementation issues surrounding LRI, incorporating active management into the solution and applying it to non-pension as well as pension challenges.

**Begin at the beginning: Portfolio optimization**
As with any investment problem, the single most useful step in figuring out how to build an LRI portfolio is
to frame the question as a Markowitz optimization problem. Markowitz or mean-variance optimization (MVO) is the procedure that finds the “best” combination of assets given estimates of their expected returns, risks, and correlations. MVO is taught in the first week of finance class and is true by construction: if the assumptions and the numerical input estimates are correct, then the resulting portfolio will be optimal. (The assumptions include, for example, the idea that investors are averse to variance of return, or volatility, rather than to some other measure or definition of risk.) By “optimal” we mean that no other mix of assets will offer a higher expected return at the same risk, or less risk at the same expected return.

We acknowledge that MVO is widely criticized. The criticisms of MVO rest on the lack of realism in the assumptions and on the difficulty of forming accurate input estimates. Because we are using MVO only as a starting point, we will not address these critiques here, but we emphasize our view that, despite any shortcomings it may have, MVO is the appropriate point of departure for all serious investment analysis.

The “liability asset” and its rate of return

We indicated earlier that the return on the liability is an important benchmark in LRI. However, the idea of the liability return is not widely understood. To begin to understand it, let’s specify that by “the liability” we mean, not the stream of expected cash flows to beneficiaries going out 50 or more years in the future, but the present value of that stream. (See our earlier paper, “Back to Basics,” for a description of the stream of expected cash flows.)

A present value is a price, a dollar amount. The present value of the liability – henceforth just “the liability” – is then the price at which a rational and fully-informed investor would be willing to assume the liability today by promising to make all of the future payments. By mentally framing the liability as a security that can be traded, one can envision the liability in the same “space” as any other asset, such as a stock, bond, or portfolio of stocks and bonds.

To incorporate the liability in an investment strategy, we must have a “model” of it. In other words, the liability must be able to be described as a portfolio of assets that can be purchased. An example would be:

\[
\text{Liability asset} = \left[0.30 \times [\text{30\% nominal bonds with a duration of 18 years}] \right] + \left[0.70 \times [\text{70\% real return bonds with a duration of 18 years}] \right]
\]

We call this hypothetical portfolio the “liability asset” to distinguish it from the liability itself, which contains risks that cannot be hedged by any existing asset. These unhedgeable risks include items such as longevity risk – the risk of the pension beneficiary pool living longer than expected – and cannot be ignored, but since we cannot hedge them in the market, we must use other tools, such as increased pension contributions, to reduce or eliminate the impact of such risks on the pension sponsor’s ability to meet its obligations.

The return on the liability is, then, simply the period-to-period return on the liability asset, plus an error term for unhedgeable risks. If nominal interest rates rise from 3% to 5%, and real interest rates (the stated interest rate on a real return bond) rise from 1% to 2%, then the return on the above described portfolio is approximately:

\[
\text{Liability return} = 0.30 \times [-2 \times 18\%] + 0.70 \times [-1 \times 18\%] = -23.4\%
\]

(Note that rising interest rates are good for the pension plan – the liability has shrunk by 23.4%. Falling interest rates are bad for the plan. It may be helpful to think of the liability as an asset held short – the bigger it gets, the worse off you are; the smaller it gets, the better off you are.)

Optimization including the liability

Figure 1 shows the usual MVO diagram, with one extra item: the pension liability (L) is shown as an asset held short, as discussed above. (A short position in an asset has a negative expected return – that is why L is below the zero line – but it still has positive risk.) Cash, denoted by r, for “risk-free,” is in its traditional place as the only riskless asset. The familiar efficient frontier of all risky assets – in this case, all assets other than cash
– is the curved line above the points $A_1$, $A_2$, etc., where each point denotes a risky asset class such as stocks, bonds, real estate, or commodities.

**Figure 1**
Efficient frontier showing assets and liability

The curved line represents the best portfolios that can be obtained by mixing only risky assets, but that is not the best that we can do. By mixing riskless cash with portfolios of risky assets, we identify the capital market line (abbreviated CML), which is superior, in terms of expected return per unit of risk, to any portfolio that is composed purely of risky assets (except at the point of tangency in Figure 1, described below).

Geometrically, the CML is identified by connecting the point representing cash, $r_f$, with the point of tangency between the straight and curved lines. The point of tangency is labeled portfolio $Q$ (this will come in handy later). Note that the CML extends to the right of portfolio $Q$; points on the CML to the left of portfolio $Q$ represent portfolios that mix cash with risky assets, while points on the CML to the right of portfolio $Q$ represent portfolios that are leveraged by borrowing cash (that is, by holding negative amounts of cash). Thus, the CML represents the best portfolios that can be obtained by mixing risky assets with either a long or short position in cash.

Classic portfolio theory, sometimes called “modern” portfolio theory, says that investors are interested in holding only portfolios on the CML. In the remainder of this paper we will say something similar but different, where the difference arises from the presence of the liability and the need to hedge it.

**How the liability changes the optimization result**

Now, let’s incorporate the liability. Let’s start by noting that if the pension fund has a duration of, say, 18 years (see the previous paper), then holding cash is not riskless. Interest rates will vary over any long period and a pension fund invested in cash might very well experience a shortfall relative to a liability with an 18-year duration. But a bond with an 18-year duration is much closer to being riskless, if one has this liability; the bond thus seems to belong in the riskless-asset position in the diagram, so that’s approximately where we’re going to put it (but not quite). The bond is not quite riskless for an investor with a liability, because duration matching is never exact and, more to the point, liabilities can vary in ways that have nothing to do with the bond market. (An increase in life expectancy would be one such source of variation.) As a result, there is always some remaining risk that is unhedgeable.

Figure 2 shows the optimization problem solved for a pension fund with liability $L$ (that is, where the liability has the expected return and risk denoted by point $L$ in Figure 1). We start by moving the entire diagram downward to account for the fact that we are subtracting the liability from the assets to arrive at a net-assets measure which, in accordance with pension convention, we call the “surplus,” even though the surplus is likely to be a negative number (a deficit). In fact, we assume throughout the analysis that the surplus, assets minus liabilities, is negative.

**Figure 2**
Assets and liability in Figure 1 reoptimized with liability treated as a short position
The least-risky combination of assets and liabilities is thus shown as \( r_S \), denoting the riskfree rate \( r_f \) as applied to the surplus \( S \). This combination, the “liability asset” minus the liability itself, is shown as being roughly where cash was in Figure 1, but farther to the right because it is not quite riskless. (For reference, the return on the “liability asset” alone, before subtracting the liability, is shown as the point \( r_{LA} \).) Moreover, it is much lower down on the diagram: the rate of return on the surplus is assumed to be slightly negative because the surplus is assumed to have a negative dollar value (the plan is assumed to be slightly underfunded). In other words, the plan has “borrowed” the deficit and is paying interest on it.³

Portfolio \( Q' \), the new tangency portfolio, is a mix of risky assets (now computed net of the liability, as are all other points on the diagram) that differs somewhat from portfolio \( Q \). (We can tell that its contents differ because it is in a different place on the diagram.) But what is in portfolio \( Q' \)?

**Next steps**

A full optimization, one that includes in the opportunity set every available asset in the world, plus the liability as an asset held short, would give the contents of portfolio \( Q \) as an output. However, as anyone who has worked with optimization software will quickly point out, the data requirements for such a task are overwhelming and the results are likely to be very imprecise. Without a shortcut or set of simplifying assumptions, one might as well not bother.

Readers who are thinking one or two steps ahead may have already figured out that the simplifying assumption most likely to be helpful in this situation is to assume that the contents of portfolio \( Q \) are the “market portfolio” of William Sharpe’s Capital Asset Pricing Model. This is the capitalization-weighted portfolio of every risky, tradeable asset in the world. The remainder of our analysis, in the next paper – called “The Liability-Relative Solution” – derives the contents of portfolio \( Q' \) (as contrasted with portfolio \( Q \), the market portfolio) and will proceed from this point of departure.

---


---

¹ For brevity, we refer to nominal bonds simply as “bonds,” and inflation-indexed bonds as “real return bonds” (RRBs).

² By “bond” in this section we really mean “mix of nominal bonds and real return bonds (RRBs).” For now, we will skip the analysis that separates bond risk into real interest rate risk and inflation risk, and that thus calculates the correct mix of nominal bonds and RRBs for hedging a given liability.

³ Sticklers will note that Figure 2 is imprecisely drawn because it mixes pure returns (such as \( r_{LA} \)) with dollar-returns, or returns multiplied by the number of dollars invested (such as \( r_S \)). Thus, the figure should be interpreted as a conceptual illustration, not a template for exact problem-solving.
On Point

The Liability-Relative Solution

SUMMARY:
Liability-relative investing, defined in our previous paper, is fleshed out in this essay. Using the familiar Markowitz optimization diagram, we draw a capital market line – the line representing all desirable portfolios – by putting the liability-hedging portfolio of assets, instead of cash, in the riskless-asset position. As a simplifying assumption, we then suppose that Sharpe’s Capital Asset Pricing Model is true, so that (in the absence of liabilities) the risky-asset position, or point of tangency between the capital market line and the curved efficient frontier, is occupied by the capitalization-weighted world market wealth portfolio of all risky assets. However, our special liability-relative capital market line passes through a different tangency point, one consisting of the world market portfolio minus those assets needed to hedge the world’s pension liabilities. Finally, we argue that the hedging properties of the liability-hedging asset portfolio should be preserved all the way up the capital market line, as the pension sponsor takes equity risk and other market risks; a derivatives overlay is needed to accomplish this objective.

Introduction
In our previous paper, entitled Beyond the Basics, we defined liability-relative investing (LRI) as the solution to a Markowitz or mean-variance optimization problem where the liability is treated as an asset held short. Solving this problem produces a capital market line (CML), or set of desirable portfolios. The CML starts at the lowest-risk end with a portfolio of nominal bonds (here called just “bonds”) and real return bonds (RRBs) that hedges the interest rate risks in the liability as much as possible. This portfolio is called the liability asset (LA).

As one climbs the CML, risky assets are added. Riskless interest rates are generally considered to be too low to support a pension program, so most pension managers invest quite a bit in risky assets – 60 to 70 percent is conventional. The expected returns on these risky assets help pay for the plan if the expectations are realized. If the market disappoints, however, the sponsor must make up the difference through additional funding. (This is why it is called “risk.”) The decision of how much to invest in risky assets depends on:

(1) the financial health of the sponsor, because only sponsors with strong balance sheets are in a position to bear a lot of market risk;

(2) the risk tolerance of the sponsor, in a psychological or behavioural sense; this corresponds roughly to how comfortably the pension executives, on one side, can ask for additional contributions if market returns are poor, and on the other side, how willing the operating executives are to provide such contributions; and

(3) the sponsor’s view of how rewarding and how risky markets are likely to be, given their pricing and other characteristics at a given point in time.

Toward the end of our earlier paper, we began to describe how we would determine the contents of the riskless and risky asset portfolios. We noted that the Capital Asset Pricing Model is a simplifying assumption that lets us determine the contents of portfolio Q, the risky-asset portfolio in our earlier analysis, and – through additional manipulation – the contents of portfolio Q1, the risky-asset portfolio after removal of bonds and RRBs needed for liability hedging. Here, we finish this thought and put some numbers (rough approximations of course) to the portfolio weights.

Finally, at the end of our earlier paper, we said that the liability interest-rate and real-interest-rate hedges should be kept in place no matter where the selected portfolio lies on the efficient frontier; liability hedging is not just for risk-averse investors adhering to an LDI or fixed-income investment policy, but for everybody. We develop this theme further here.
The CAPM and the composition of portfolios \( Q \) and \( Q^1 \)

So far, we have established the purpose and general nature of LRI: we want to be somewhere – that is, at some risk level – on a special CML, carefully defined so that the liability-hedging asset portfolio, \( L_A \), is considered the (almost) riskless asset; see Figure 1. However, we have not provided much in the way of specific portfolio construction advice. We now use the Capital Asset Pricing Model (CAPM) of William F. Sharpe and others\(^1\) to simplify the portfolio construction problem to a point where we can build a solution out of index funds and relatively simple derivative positions. We understand that almost all pension managers will want to enhance this solution by adding alpha, but that discussion is reserved for the next paper.

Figure 1

![Capital Market Line](image)

Like mean-variance optimization (MVO), which was discussed in detail in our previous paper, the CAPM is widely criticized for its assumptions, but we will not get very far in figuring out the composition of portfolio \( Q^1 \) without it. (Specifically, unless we can make some dramatic simplifying assumptions, we would have to run an optimizer using estimates of the expected return and risk of every asset in the world, and the correlation of each asset with every other; this is, to put it simply, impossible.) Assuming that the CAPM is true turns out to be immensely helpful, much more helpful than any other simplifying assumption we can think of.

The CAPM holds if: (1) all investors see the same “picture.” That is, every investor has the same estimates of expected return, risk, and correlation for every security or security pair; (2) all investors optimize; and (3) all investors can borrow and lend as much cash as they want at the riskless rate. If these conditions hold, then the only portfolio of risky assets that any investor would want to hold is the “market portfolio,” which consists of a share or slice of all of the risky assets in the world, each held in proportion to its total market capitalization. “Risky” here means “not cash.” In other words, portfolio \( Q \) is the cap-weighted world market portfolio of risky assets. The CML then consists of mixes of portfolio \( Q \) and positive or negative positions in cash, where a negative or short position in cash means borrowing to buy extra units of portfolio \( Q \). This relationship is not shown in Figure 1, but has a similar appearance, with cash (not the liability-hedging asset portfolio) in the riskless position and with portfolio \( Q \) (consisting of all risky assets, not just those left over after liability hedging) at the point of tangency between the CML and the efficient-frontier curve.

Note that the cap-weighted world market portfolio is the only portfolio that is macroconsistent. This means that if every investor in the world decided to hold it, the prices and quantities of the assets that currently exist in the world would not have to change.

Portfolio \( Q \) doesn’t consist just of equities. All investable, risky assets are in it. Figure 2 gives a rough estimate of the asset-class weights in portfolio \( Q \). The estimates for global stocks and bonds are accurate, but those for global real estate and commodities are very approximate because no one has precisely measured these quantities.

Figure 2

![World Market Wealth Portfolio](image)
slightly different. Following Waring and Whitney (2009), we recall that the almost-riskless asset for an investor with a liability is portfolio LA. In Figure 1, we graph portfolio LA in the position of an almost riskless asset—“almost” because there are some risks in the liability, such as longevity risk, that are unhedgeable in the capital markets.

Next, we remove the assets that make up portfolio LA from portfolio Q to avoid double-counting (that is, double-holding) them, thereby creating a new “market portfolio,” $Q'$, that is composed of all the risky assets in the world not needed to hedge pension liability risk. Portfolio $Q'$ in Figure 1 thus contains equities, real estate, commodities, credit bonds, and foreign sovereign bonds and RRBs, but not the domestic bonds and RRBs that go toward making up portfolio LA. It would also contain human capital if it were possible to invest in it, but it is not possible. Finally, an argument can be made for including cash in portfolio $Q'$. Suggested asset class weights for portfolio $Q'$ are in Figure 3.2

Liability hedging all the way up the capital market line

But we are going to abandon this conventional practice (of adding to 100%) because it keeps us from properly hedging the interest rate exposures in the liability when we also take market risk. (Market risk, the risk of a cap-weighted portfolio of all assets not in portfolio LA, is roughly proxied by equity risk, and it is often helpful to think of it as just equity risk even though real estate, commodities, and certain bonds are in it.) If hedging all the hedgeable risks in the liability is a good idea when we are not taking equity risk, it’s an even better idea when we are! But, in a portfolio with a 100% adding-up constraint, as we add equities and other risky assets, they displace the bonds and RRBs that provide the hedge. We therefore use interest-rate derivatives, which do not consume capital (they only require a small variation-margin deposit and periodic adjustments), to fill in the part of the liability hedge that is not provided by direct bond and RRB holdings. At this stage, where we have no active management, the pension portfolio consists of:

- Index funds providing exposure to portfolio $Q'$, in an amount from 0% to 100% of capital invested;
- Bonds and RRBs providing some or all of the liability hedge (all of it if the allocation to portfolio $Q'$ is zero, otherwise some of it)
- An interest rate derivatives overlay providing the rest of the liability hedge.

The exact composition of the derivatives overlay is beyond the scope of this paper, and depends on institutional arrangements in the jurisdiction where the investor resides or does business. It suffices to say that the overlay, when combined with the “physical” (non-derivative) bond and TIPS positions, is designed to fully hedge the liability in terms of its sensitivity to interest rate movements. To achieve this, the assets, including both derivatives and physicals, must have the same dollar duration as the liability, where dollar duration is defined as the duration of the asset or liability multiplied by the number of dollars invested in the asset or liability. (A $1 million liability with a duration of 18 years thus has a dollar duration of 18 million.)

The nominal-interest-rate hedge may be further refined into a real-interest-rate hedge and an inflation hedge, reflecting the fact that the value of a pension liability changes by a different amount when a nominal interest
rate change is caused by a real interest rate change than when it is caused by a change in inflation. The technology for doing this is in Waring (2004), with a simplified explanation in Siegel and Waring (2004), and involves allocation between nominal bonds (and derivatives thereof) and RRBs (and derivatives thereof).

Choosing a portfolio on the capital market line

We are about done. We need to choose a portfolio on the with-liability capital market line in Figure 1. Utility theory provides a method of doing so, but we are not particularly impressed with this method. It requires knowing the utility function of the investor. The utility function is, essentially, a schedule of rates at which the investor is willing to “buy” a higher than expected (but not in any way guaranteed!) return by taking more risk. (That is, by making the sacrifice of being willing to accept a lower return if markets disappoint). This rate is almost impossible to estimate numerically.

Therefore, we prefer the “financial planning” method that backs into the preferred risk-return tradeoff by simply asking how much risk the investor is willing to take. The investor, in this case the pension plan sponsor, chooses the point on the capital market line that corresponds to the desired, or perhaps we should say tolerated, standard deviation. Since each point on the capital market line represents a specific portfolio of risky assets (those constituting portfolio Q1) and liability-hedging assets (including both physicals and derivatives), we have a solution to the pension investor’s problem. For reasons explained earlier, the solution is referred to as an LRI solution. An example, showing portfolio holdings, was presented in Figure 2.

The investor must then decide whether to try to enhance the portfolio returns by adding alpha. Alpha may come from anywhere. Among the potential sources of alpha in this portfolio structure are: changing the weights of the asset classes in portfolio Q1 or in the liability hedge; active management within one or more asset classes; and other sources, such as portable alpha in the form of, say, a market-neutral overlay. Our next On Point paper, “Adding Alpha to the LRI Solution,” covers these issues.
Let’s Start with LDI: The Long Bond Solution

Recall that pension liabilities have a long duration, and are also typically exposed to inflation risk (the latter if there is a cost of living adjustment, or COLA, in the pension promise). A bond portfolio with a long duration is matched to the liability and has minimal risk. The BMO Asset Management Canadian Long-Bond Alpha Fund is one such solution, specifically designed to hedge the risks of pension liabilities. The Fund combines a long duration fixed-income index fund with a partial allocation to an equity market-neutral strategy (BMO Asset Management Canadian Pure Alpha Fund) to create a more effective portfolio structure for pension clients. The alpha component helps to hedge inflation risk, which otherwise could only be hedged directly by holding very low-yielding real-return bonds.

Broad asset class exposure – seeking global beta

An investor wanting to take more than the minimum amount of risk – and most investors will fall into this category – could invest first in promising asset-class exposures, as they move along the spectrum towards alpha opportunities. While taking more risk means that a higher long-term return can be expected, one should not simply pile on risk in the hope of an excellent return. Risk is called risk because it has a downside – additional
required cash contributions to the pension fund. The amount of asset-class risk (market risk) taken should be commensurate with the sponsor’s ability to make such contributions at times when the market has been the most unrewarding.

Equity index products can provide much of the desired exposure to risky asset classes. These include BMO’s line-up of equity ETFs. The ETFs needed for constructing the equity part of a broad asset class portfolio are:

- BMO S&P/TSX Capped Composite Index ETF
- BMO U.S. Equity Index ETF
- BMO International Equity Index ETF, and
- BMO Emerging Markets Equity Index ETF

with each fund weighted in proportion to the market cap of the country or countries represented by the fund.2

**Real estate, commodities, and other asset classes**

As we pointed out in “The Liability-Relative Solution,” equities are not the only risky asset that investors seeking diversification should hold. Among the others are real estate and commodities. The BMO Equal Weight REITs Index ETF is a real estate index fund that holds Canadian real estate investment trusts (REITs), and forms a part of the optimal portfolio as we’ve defined it. However, Canadian real estate is only a small part of the world’s investable real estate. A global portfolio of real estate securities could provide much more diversification.

Commodity index products (ETNs) provide passive exposure to the commodity asset class. A great many actively managed commodity indices are also available. Other asset classes that may belong in the risky-asset portfolio of an investor following our advice include certain types of bonds. The analysis of which bonds to hold should be done on a customized basis. Among the bonds that almost any investor should consider are:

- Investment-grade corporate bonds (global)
- High-yield bonds (global)
- Emerging market debt

A bond specialist can recommend other types of issues and funds. For example, in the wake of the crash of 2008, some specialists are managing portfolios of mortgage-backed securities acquired at distressed prices. These portfolios are lucrative as of this writing, but the opportunity will fade over time. Such an investment strategy is rarely amenable to indexing and belongs in the category of “adding alpha,” to which we now turn.

**Adding alpha**

Adding alpha to an already optimal blend of beta exposures (asset classes and say, duration extension overlays) is mostly a matter of selecting winning active managers. Another way to add alpha is to vary the beta exposures to time the market or to achieve an overall payoff that the investor believes will be better than the objectively optimal blend.

**Selecting winning managers and funds**

In practice, sponsors may forego indexing entirely and build the portfolio out of active managers. This means that selecting beta exposures and adding alpha are accomplished in one practical step, although the single step involves two conceptual sets of decisions (beta decisions and alpha decisions).

**Quantamental strategies**

The investor seeking to place a set of winning alpha bets while simultaneously achieving the desired beta exposures should consider BMO Asset Management’s Quantamental strategies, blending quantitative and fundamental analyses. These strategies have many of the attributes of quantitative management:

- Rigorous and disciplined investment processes
- Broad opportunity set
- Explicit risk management

while capturing the advantages of fundamental analysis:

- Deep economic insight
- Proprietary data and investment insights
- Individual company analysis
- Focus on forward-looking metrics
- Direct security selection input from fundamental research analysts

Our belief is that an approach offering the best of both styles has the highest probability of achieving a consistent positive alpha and of maximizing the information ratio.

The Quantamental funds that are relevant to solving the liability-relative investor’s problem are:
• BMO Asset Management Canadian Core Alpha;
• BMO Asset Management Canadian Pure Alpha (which is an equity market neutral strategy)
• BMO Asset Management Liability Sensitive Equity Fund (more about this later, since it occupies a special place in an LRI strategy)

The Canadian Core Alpha Fund offers a two-fold approach:

1. A focus on capturing insights at the sector level – a crucial element for a Canadian strategy, because industry and sector differences are particularly important in this market.
2. A framework and process for collecting and managing data in order to generate new investment insights. The intent is to offer consistent and reliable alpha with active-risk control, thus improving the information ratio or return per unit of risk taken.

The Canadian Pure Alpha Fund uses long-short active management to capture alpha insights from the Quantamental process, holding both long and short positions to manage risk. It is intended to offer investors uncorrelated alpha and positive absolute returns in a variety of market conditions.

**Traditional fundamental strategies**

In addition, BMO Asset Management offers fundamental equity strategies (large cap, small cap, dividend, global, etc.) that are also designed to generate alpha.

**Earning alpha by holding unusual asset class weights or by varying the weights over time**

In general, market timing and other alpha-generation techniques apart from selecting winning managers are beyond the scope of this paper. However, we’d like to give one timely example. If the investor has a long-term strategic view that interest rates are going to rise, he or she may wish to forego duration matching (either through direct investments in bonds or through an overlay) until rates actually rise. Such a strategy avoids locking in today’s rock-bottom interest rates, and enables the investor to earn much higher rates in the future if rates do in fact rise.

Of course, the investor might be wrong and rates might not rise, or they might fall further causing the liability to increase beyond current levels. Note that waiting to invest in bonds is not a “free” option. It has an opportunity cost, namely the risk inherent in running the pension fund with no liability hedge while waiting for bond yields to rise. If yields instead fall further, the plan could be in deep trouble.

**Investing in liability-sensitive equities**

The Holy Grail of liability-relative investing is a portfolio that both delivers a liability hedge (in the dual dimensions of real interest rate risk and inflation risk) and captures the equity risk premium, plus any alpha the manager is able to earn. To help mitigate a pension liability, such a portfolio would need to have a long duration; not in the sense of a long investment time horizon, but in the bond-market sense of rising substantially when interest rates fall and falling when rates rise. The BMO Asset Management Liability Sensitive Equity (LSE) Fund is engineered to achieve this very attractive set of payoffs.

The LSE Fund is constructed by favoring – that is, adding to the optimization function – a liability beta, or factor that represents each stock’s correlation with a long-term Canadian liability index (the DEX Long Federal Index), and by minimizing equity beta risk subject to the liability goal. BMO Asset Management also uses its stock-selection process to further refine the portfolio in the hope of earning an alpha above what the first two objectives provide.

The resulting portfolio has a beta relative to the S&P/TSX Composite of 0.6 to 0.8, and a materially higher dividend yield. While BMO Asset Management does not specifically tilt the portfolio toward dividend yield, the liability beta (each stock’s sensitivity to the DEX Long Federal Index) is highly correlated with yield and provides a nice yield pick-up over the index. The LSE Fund thus looks like a low-volatility fund, but with a slightly higher correlation to interest rates.

Because there are no stocks that behave exactly like a bond in terms of their interest rate sensitivity, it is not possible to get a high R² (a measure of closeness of fit) between an equity portfolio, no matter how well designed, and a pension liability. That said, the LSE portfolio is much better than the standard market capitalization based indices with regard to interest rate sensitivity. Exhibit 1 compares the total return indices of the S&P/TSX Composite Index, the BMO Asset Management LSE Fund, and the DEX Long Federal Index. As you can see, the LSE Fund is less volatile than stocks and has a narrower spread band.
As you can see, the LSE Fund is one-third less volatile over the long term than the broad equity market and has an improved correlation to long bonds versus the S&P/TSX Composite Index.

**Hedging the liability all the way up the efficient frontier**

We noted in our earlier articles that the portfolio should be hedged to the liability all the way up the efficient frontier. That is, the total duration of the portfolio should match that of the liability, no matter how much of the portfolio is invested in equities, real estate, commodities, and other risky assets.

At low levels of risky-asset exposure, this hedge can be achieved simply by lengthening the duration of the bond portfolio. If, for example, the duration of the liability is 15 years, an asset mix consisting of 100% in bonds should also have a duration of 15. If the asset mix is only 50% in bonds, however, a duration of 30 is required for the bond portion.

However, bonds with a duration of 30 years may not exist. The longest corporate bonds have maturities of about 40 years, and the longest government bonds around 30. Since the duration of a coupon-paying bond is less than the maturity, there is a practical limit to how much duration extension (and thus full liability hedging) can be achieved by lengthening the duration of the bond part of a multi-asset-class portfolio.

Beyond that practical limit, a derivatives overlay must be used. The most common overlay is a swap agreement. The investor delivers the cash flows on a floating-rate instrument and receives the cash flows on a very long-duration, fixed-rate instrument. Such a position does not require much cash collateral and can thus be held in addition to the risky asset classes in a portfolio.

In previous work, we’ve referred to the overlay strategy as achieving a more than 100% invested position. This can be scary. Investors are generally wise to be concerned about leverage and derivatives. However, when either leverage (investing more than 100% of capital) or a derivative position is really used to reduce risk, those uses are an appropriate and valuable application of these tools.

**Foundations and Endowments**

So far, we have concentrated on investment strategies for defined-benefit pension funds for two reasons:

(1) in the past, that is where the money was, so the greater part of the effort expended in developing investment strategies was for those funds; and

(2) defined-benefit pension funds have easily measured liabilities.

We now turn to investors whose liabilities are not as easily defined.

Foundations and charitable endowments (universities, churches, museums, hospitals, etc.) often enjoy great latitude as to how much of their assets they can spend, and may not think of themselves as having a liability at all. In Canada, only private foundations face a spending minimum (4.5% of then-current asset value each year), and other endowed institutions may generally spend as much or as little as their trustees choose.

But this does not mean that endowed institutions are free of liabilities. In fact, the liability may be thought of as the present value of all future spending. In an accounting sense, assets must equal liabilities plus owner’s equity.
for any organization, and there is no owner’s equity in a foundation, university, or church – the trustees cannot vote to close the institution and keep the money! As a result, the liability is equal to the assets.

For an organization that gives away 4 to 5% of then-current assets each year, into perpetuity, the “liability” duration is about 30.5 (We use quotes since it is not a legal liability.) This organization consequently faces a great deal of interest-rate risk, much more than any pension fund. It thus seems prudent for an endowment, foundation, or other institution with an annual payout rate in the range of 4 to 5% to be very aggressive in managing interest-rate risk (duration risk). This can be achieved through the derivatives overlay described earlier.

A lower payout rate means an even longer duration, but also less operating risk (because the institution holds onto more of its money). A higher payout rate, while shortening the duration in a technical sense, means that the institution should worry even more about running out of money, and should be managing its risks, including interest-rate risk, with a short rope.

Most endowed institutions act as if they have no liability and are asset-only investors. They typically load up on equities, hedge funds, and private securities and shun bonds. They also do not usually use duration-extending derivatives overlays. Our analysis suggests that risk management for endowed institutions is incomplete – that they are massively exposed to interest-rate risk, like a pension fund that has not bothered to liability hedge. They would probably benefit from a closer examination of liabilities.6

**Insurance Companies**

Insurance companies, likewise, manage asset pools that are intended to pay future liabilities. The balance sheet of an insurance company is primarily made up of investment assets on the asset side and actuarial reserves on the liability side. Although life insurers and property-casualty insurance firms have quite different formal requirements, the investment policies of both types of firms emphasize fixed income over equities.

For the sake of simplicity, let’s focus on life insurers. We think of these firms’ balance sheets as being liability-driven; the amount of assets they have is a function of the need to support the future liabilities they created from making pay-out promises to customers.

Any insurance contract anticipates a possible pay-out. Insurance companies must therefore ensure that the contract is appropriately priced so that the sum of the premiums received and the investment returns earned will be larger than the actuarially forecasted pay-out.

As recent events have shown, some insurance companies aggressively priced their premiums to capture market share without fully matching the cash flows from their investments to their anticipated liabilities. They instead hoped that high market returns would produce the needed profits. Such companies paid a high price for their risky behavior because, after a long period of disappointing market returns, their reserves turned out to be underfunded. In the future, they should pursue liability-relative investing practices.

**Individual Investors**

The liability for an individual investor consists of his or her spending plans – more precisely, the present value thereof. For example, a retiree who needs to generate $100,000 per year in income, in real (inflating) terms for the rest of his or her life, can perform asset-liability analysis on that stream of projected or intended cash flows. If the investor’s portfolio performs better (or worse) than expected, or if the investor saves more (or less) than he was planning, then the asset-liability analysis needs to be adjusted accordingly.7

Like pension funds and endowed institutions, individuals saving for retirement face a great deal of interest-rate risk, which can be hedged through a duration-extension strategy. Such strategies have not proven popular with individual investors, perhaps because it is hard for them to visualize how the fairly abstract concept of interest-rate or duration risk translates to an actual problem for the investor. Let’s illustrate it using annuity rates: a 65-year old male saver who accumulates $1,000,000 can afford to buy an immediate life annuity paying $90,000 per year if long-term interest rates are 6%, but only $40,000 per year if long-term interest rates are 2%. Now, that’s risk!

Thus, individuals should hedge the risks of their liabilities just like anyone else.
Conclusion

Hedging long-term liabilities, whether those of a pension fund, a different type of institution, or an individual, is best accomplished using long-term bonds (and long-term inflation-indexed bonds if the liability includes a cost of living adjustment). The BMO Asset Management Canadian Long-Bond Alpha Fund is one such turn-key solution. Investors wishing to take risk to add return (the realization of which is likely but not guaranteed) can invest in equity, real estate, and commodity index funds as noted earlier. When compared to a liability benchmark, the return from such a risk portfolio can be regarded as a type of “alpha.”

True alpha, however, the return from skillfully selecting securities, must be obtained through active management. BMO Asset Management’s Quantamental and fundamental strategies are a potentially productive source of alpha return from active management.

Particularly, BMO Asset Management’s Liability Sensitive Equity Fund provides all three desirable factors:

(1) the interest rate sensitivity of liabilities (to the extent that can be obtained through stock investments),

(2) equity beta, and

(3) the promise of alpha from active management skill.

We hope you have enjoyed this On Point series:

• Issue #1 Back to Basics — Why Pension Funds Exist, and How to Manage Them in the Simplest Possible Way

• Issue #2 Beyond the Basics — Pension Management using Liability-Relative Investing

• Issue #3 The Liability-Relative Solution

• And this final 4th Issue: Adding Alpha to the LRI Solution

References:


TM® Trade-marks/registered trade-marks of Bank of Montreal, used under licence

The information provided herein does not constitute a solicitation of an offer to buy, or an offer to sell securities nor should the information be relied upon as investment advice. Past performance is no guarantee of future results.

All Rights Reserved. The information contained herein: (1) is confidential and proprietary to BMO Asset Management Inc. (“BMO AM”); (2) may not be reproduced or distributed without the prior written consent of BMO AM; and (3) has been obtained from third party sources believed to be reliable but which have not been independently verified. BMO AM and its affiliates do not warrant or make any representations regarding the use or the results of the information contained herein in terms of its correctness, accuracy, timeliness, reliability or otherwise, and do not accept any responsibility for any loss or damage that results from its use.

© Registered trade-mark/trade-marks of Bank of Montreal, used under license.